
ABSTRACT

Now a day's every complex non linear mathematical problems is solved by bio inspired optimization algorithms, whether that problem is related to complex robotics system or to any electrical system to match reference value or to any filter to suppress ripples etc. In this paper the latest firefly optimization is discussed and variants of firefly with various random function for different non linear mathematical objective function is discussed. The point is proved in this paper is that the random distributed function affects the optimization.

Keywords: Matlab, Firefly, Algorithm

I. INTRODUCTION

Automatic problem solving with a digital computer has been the eternal quest of researchers in mathematics, computer science and engineering. The majority of complex problems (also NP-hard problems [1]) cannot be solved using exact methods by enumerating all the possible solutions and searching for the best solution (minimum or maximum value of objective function). Therefore, several algorithms have been emerged that solve problems in some smarter (also heuristic) ways. Nowadays, designers of the more successful algorithms draw their inspirations from Nature. For instance, the collective behaviour of social insects like ants, termites, bees and wasps, or some animal societies like flocks of bird or schools of fish have inspired computer scientists to design intelligent multi-agent systems [2]. Firefly algorithm is the latest optimization amongst these. This is based on the behavior of fireflies which beautify the sky at night with their light.

Firefly Optimization has unique randomness factor. It searches for optimal solution by considering the randomness in a constructive way. So randomness plays a very important role in exploitation and exploration in search process. The exploitation moves the fireflies in the vicinity of the promising searching space while exploration looks for new searching space. In line with this, several random distributions can be helpful. For example, uniform distribution generates each point of the search space using the same probability. On the other hand, Gaussian distribution is biased towards the observed solution, that is the smaller modifications occur more often than larger ones [1]. In this paper different noisy non linear mathematical functions are considered with different randomness function like uniform distribution and normalize Gaussian distribution random function. The paper is categorized into four main sections. First section provides introduction of the topic, second put light on the background of firefly algorithm and various random distribution function. Third one is the chapter that inherits the results for different random function for different non linear mathematical function. Although there are various random functions available for use but we have shown results for only uniform and normalize distribution random functions. Fourth chapter concludes the paper.

II. FIREFLY ALGORITHM

Firefly Algorithm (FA) was first developed by Xin-She Yang in late 2007 and 2008 at Cambridge University, which was based on their light flashing and behavior of fireflies. Firefly optimization have three considerations:

- Fireflies are unisex so each firefly will be attracted to other fireflies regardless of their gender.

- The attractiveness decreases with the distance as both are directly proportional to each other. Because of this each firefly will get attracted towards brighter firefly and if there is no brighter firefly amongst two then each will move in random direction.
- The brightness of a firefly is determined by the objective function depending upon the application.

As a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, so attractiveness β can be related with distance r as

$$\beta = \beta_0 e^{-\gamma r^2}$$

where β_0 is the attractiveness at $r = 0$.

When a firefly 'i' is attracted towards another firefly 'j' then that movement can be represented by the formula given below

$$x_i^{t+1} = x_i^t + \beta_0 e^{-\gamma r_{ij}^2} (x_j^t - x_i^t) + \alpha_t \epsilon_i^t$$

In this second term represents the attractiveness. α_t is the randomization parameter, and ϵ_i^t is a vector of random numbers drawn from a Gaussian distribution or uniform distribution at time t and γ is an absorption coefficient. If $\beta_0 = 0$, that means there is no attractiveness between any firefly and firefly will move randomly. On the other hand, if $\gamma = 0$, it reduces to a variant of particle swarm optimization. Furthermore, the randomization ϵ_i^t can easily be extended to other distributions.

2.1 Parameter Settings

α_t is randomness parameter and firefly's movement is very sensitive to it and it can be tuned during iterations with iteration counter t .

So a α_t dependent upon the iteration counter t can be expressed as

$$\alpha_t = \alpha_0 \delta^t$$

where α_0 is the initial randomness scaling factor, and δ is a cooling factor. For most applications, we can use $\delta = 0.95$ to 0.97 .

The initial scaling of firefly optimization parameters is very important as performance depends upon it. The randomness scaling parameter is initially set to 0.01 which is required for steps to reach the target without covering large distance at once since small step size will yield good results. The parameter β controls the attractiveness, and parametric studies suggest that $\beta_0 = 1$ can be used for most applications. However, should be also related to the scaling L . In general, we can set $\gamma = 1/\sqrt{L}$. For most applications, the population size $n = 15$ to 100 , though the best range is $n = 25$ to 40 . In our work it is set to 6 so that less computation time it takes.

2.2 Random Distribution Function

2.2.1 Uniform Distribution Function[2]

Uniform continuous distribution has the density function, as follows:

$$p(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Note that each possible value of the uniform distributed random variable is within optional interval $[a, b]$, on which the probability of each sub-interval is proportional to its length. If $a \leq u < v \leq b$ then the following relation holds:

$$P(u < x < v) = \frac{v-u}{b-a}$$

Normally, the uniform distribution is obtained by a call to the random number generator. Note that the discrete variate functions always return a value of type unsigned which on most platforms means a random value from the interval $[0, 2^{32}-1]$. In order to obtain the random generated value within the interval $[0, 1]$, the following mapping is used:

$$r = \frac{(\text{double})\text{rand}(\)}{(\text{double})(\text{RAND}_{MAX}) + (\text{double})(1)}$$

[IDSTM-18]

ICTM Value: 3.00

 2.2.2 Normal/Gaussian Distribution Function[2]

Normal or Gaussian distribution is defined with the following density function:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-a}{\sigma}\right)^2}$$

The distribution depends on parameters $a \in \mathbb{R}$ and $\sigma > 0$. This distribution is denoted as $N(a, \sigma)$. The standardized normal distribution is obtained, when the distribution has an average value of zero with standard deviation of one, i.e., $N(0, 1)$. In this case, the density function is simply defined as:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}$$

The Gaussian distribution has the property that approximately 2/3 of the samples drawn lie within one standard deviation. That is, the most of the modifications made on the virtual particle will be small, whilst there is a non-zero probability of generating very large modifications, because the tail of distribution never reaches zero.

III. RESULTS

The four peak function for uniform distribution random function was implemented. The four peak function's mathematical equation is given as

$$f(x, y) = e^{-(x-4)^2-(y-4)^2} + e^{-(x-4)^2-(y-4)^2} + 2[e^{-(x)^2-(y)^2} + e^{-(x)^2-(y+4)^2}]$$

The MATLAB plot for four peak function is shown in figure 3.1.

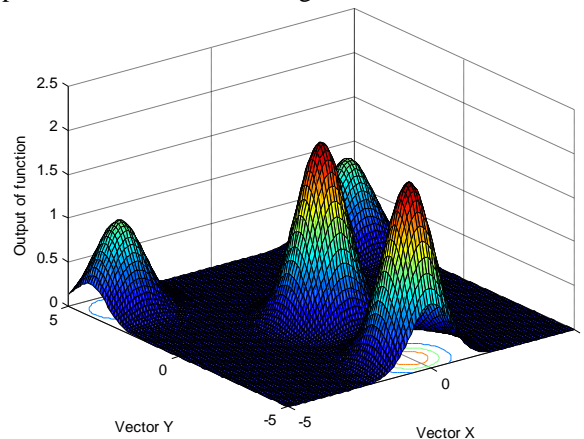


Figure 3.1: MATLAB plot of four peak non linear mathematical function

The randomness factor alpha is considered 0.1, gamma = 1 and delta is 0.97 for our results. The randomness factor can be in between [0, 1] only. If uniform distribution function is considered then the figure 3.2(a) shows the initial positions assigned to each firefly and 3.2(b) shows final position settled. In figure 3.2(a) every firefly is settled at that position where it will get attracted maximum or light intensity of other firefly is maximum. That's why figure shows all fireflies collected at one place. The plot of maximum light intensity is shown in figure 3.3. it shows that 27 iterations all fireflies are settled to optimum position and the light intensity sensed is maximum. Now if normalize random number is used then the fitness function plot is shown in figure 3.3 along with the plot of uniform random function.

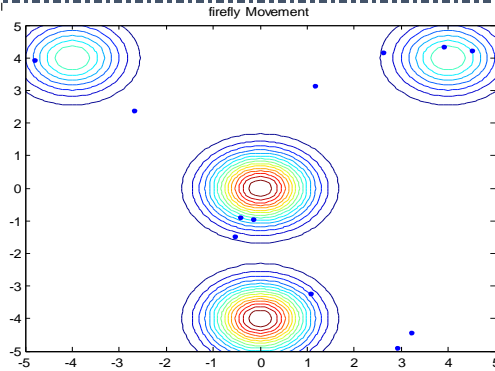


Figure 3.2(a): Initial positions of firefly

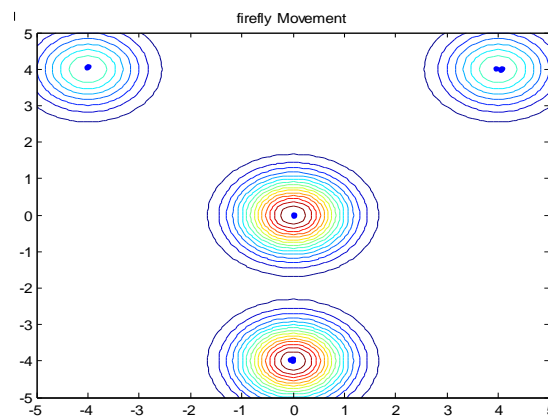


Figure 3.2(b): every firefly settled to optimum position

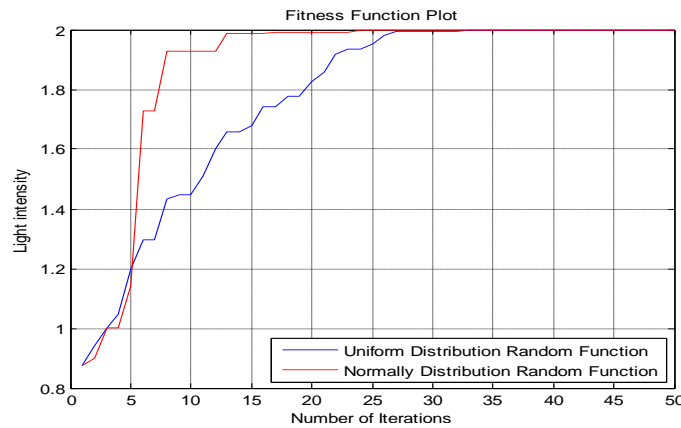


Figure 3.3: Light Intensity Plot of firefly for each iteration

Figure 3.3 plot shows that normalize random function gives better fitness function value than uniform. The time consumption for this case is shown in table 3.1.

Table 3.1: Time consumption for optimum value settlement in every case

Non Linear Function	Uniform Distributed Random number	Normally Distributed Random number
Four Peak Non linear function	3.42e-006	2.44e-006
Parabolic function	3.90e-006	1.95e-006
Rastrigin function	3.90e-006	1.95e-006

The parabolic function is used as objective function and its mathematical equation implemented is

$$f(x,y) = 12 - (x^2 + y^2)/100$$

Plot in MATLAB is shown in figure 3.4 below.

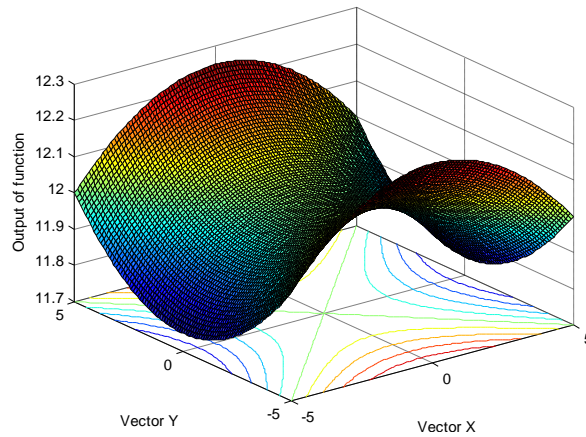


Figure 3.4: Plot of Parabolic function in MATLAB with 101 points along x and y axis.

The light intensity plot again in this case is shown in figure 3.5. in parabolic function also fitness function value comes higher for normalize random number than uniform random number. The time taken for later case is also less.

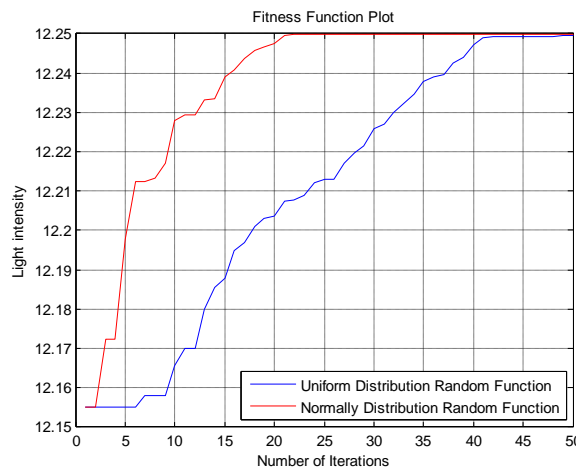


Figure 3.5: Light Intensity Plot of firefly in case of parabolic function for each iteration

Now same firefly parameters are implemented for Rastrigin function. The mathematical form of function is

$$f(x,y) = 80 - [20 + x^2 + y^2 - 10(\cos(2\pi x) + \cos(2\pi y))]$$

The MATLAB plot for this is shown in figure 3.6. In case of normalize random function the fireflies movement is shown in figure 3.7 for rastrigin function.

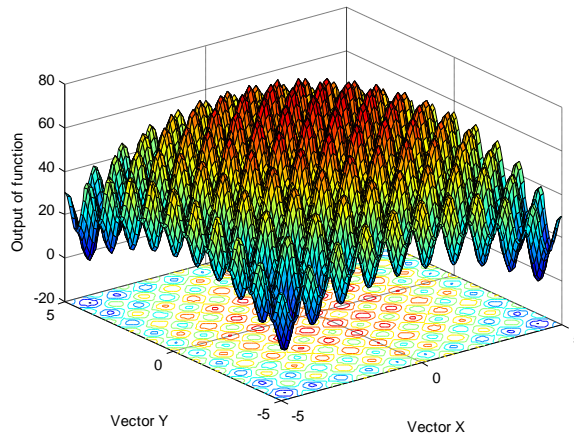


Figure 3.6: MATLAB plot of Rastrigin function

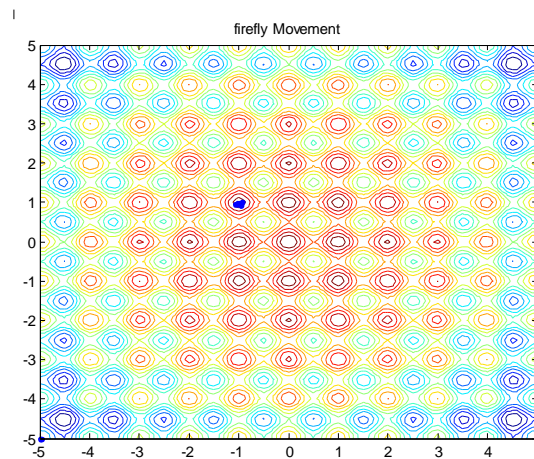


Figure 3.7: Rastrigin function's firefly movement

The fitness function comparison for uniform and randomize function is shown in figure 3.8.

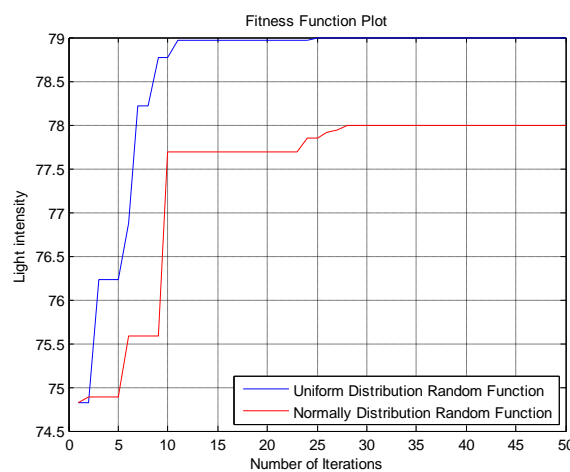


Figure 3.8: Fitness function for Rastrigin non linear objective function

In this plot the fitness function values come out to be less in case of normalizing random functions than uniform. A comparison bar graph is plotted in figure 3.9 to shows the time consumption between both random functions.

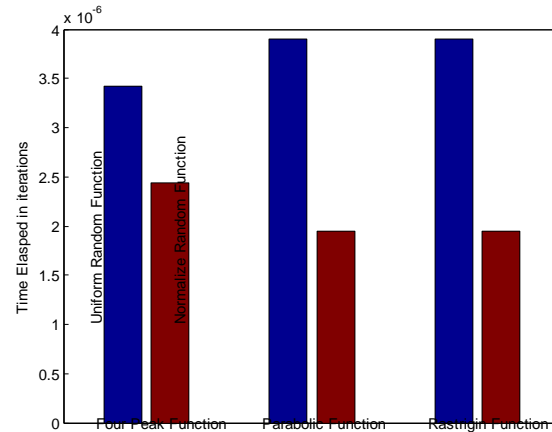


Figure3.9: Time elapsed between different functions

The blue bar in above figure is for uniform random function and red is for normalizing random functions. This graph gives a clear visualization that normalize random function gives better time saving than uniform and previous discussion has also proved that fitness function value is also better in normalize random distribution function. Thus this is the way how random factor changes the property of firefly algorithm.

IV. CONCLUSION

In this paper we have discussed the firefly optimization. In the position updating formula, a factor which is sensitive to randomness of fireflies is mentioned. As discussed randomness has a quite significant effect in the optimization to get the optimum value. So this paper discussed the two mainstream random functions and their effect on various non linear objective functions. It has been observed that normalize random distributed function performs better than uniform random distribution function in terms of time consumption and fitness function value except for Rastrigin function

V. REFERENCES

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